2018 IEEE Wireless Communications and Networking Conference (WCNC)

Adaptive Successive Transmission in Virtual Full-Duplex Cooperative NOMA

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***Abstract*—In this paper, we propose a novel *virtual* full-duplex cooperative non-orthogonal multiple access (NOMA) technique for a downlink two-hop cellular network which consists of a single base station (BS), two mobile stations (MSs), and K half-duplex decode–and–forward (DF) relay stations (RSs). In the proposed technique, the BS sends super-imposed signals for two MSs via the RSs in each transmission phase, while a selected RS via two-stage relay selection sends the signals to two MSs. Thus, the multiplexing loss due to half-duplex operation of RSs can be overcome by allowing for both the BS and a selected RS to send data at the same time. In the proposed cooperative NOMA, we *adaptively* reset the successive transmission according to decoding status at RSs. As main results, we mathematically analyze outage probability and diversity-multiplexing tradeoff (DMT) of the proposed technique. Extensive computer simulations show that the proposed technique significantly outperforms the existing schemes in terms of both outage probability and DMT.**

I. Introduction

Non-orthogonal multiple access (NOMA) is considered as a promising candidate of 5G technologies [1] in order to meet the requirements of 5G, massive connectivity, low latency, high spectral efficiency, etc [2]. The merit of NOMA comes from the capability of support for multi-user utilizing a single domain, *i.e.,* power, code domain [3]–[6].

Based on the existing conventional NOMA, various cooperative NOMA systems have also been focused due to the additional spatial diversity through relay communications [6]–[9]. The authors in [6] coped with relay selection for NOMA in the presence of multiple half-duplex decode-and-forward (DF) relay stations (RSs) and analyzed the proposed two-stage relay selection scheme in terms of outage probability. The work in [7] introduced NOMA in coordinated direct and relay transmission (CDRT) in order to improve the user having relatively poor channel where one user communicates with a base station (BS) directly, whereas the other user can receive message from the half-duplex DF RSs. In [8], a cooperative NOMA scheme in which two half-duplex DF RSs forward the messages to two users simultaneously using dirty paper coding was analyzed in terms of outage probability. Achievable rate of cooperative half-duplex relaying scheme for NOMA was also analyzed using efficient approximation via Gauss-Chebyshev Integration in Rician fading environments [9].

However, aforementioned works inherently suffer from a multiplexing loss since half-duplex RSs cannot transmit and receive simultaneously. That is, at least two transmission phases are necessary to transmit a signal to users in the absence of channels between a BS and users. Motivated on compensating a multiplexing loss due to half-duplex RSs, we consider a downlink cooperative NOMA scenario and propose spectrally efficient multi-hop successive relaying with multiple half-duplex RSs based on the proposed two-stage relay selection for virtual full-duplex (VFD) operation. In the proposed protocol, since inter-relay interference exists due to successive relaying, it is critical how to deal with inter-relay interference. Hence, we design the dynamic inter-relay interference management based on [10], which is the scheme known as achieving best diversity-multiplexing tradeoff (DMT) in the DF half-duplex multi-relay scenario without a direct link to the best of our knowledge so far.

In this paper, we propose the protocol that can operate VFD with multiple half-duplex DF RSs leveraging dynamic inter-relay interference management in the downlink cooperative NOMA environments. The proposed protocol is based on the two techniques: two-stage relay selection and adaptive reset. For relay selection, RSs which will not be selected are weeded out via two conditions. Adaptive reset enables a restart of the transmission protocol even if there exist candidates of relay selection in order to enhance diversity gain in the moderate and high multiplexing gain regime. We analyze the proposed protocol in terms of both outage probability and DMT, and obtain the closed form exploiting Markov chain in order to resolve the dependency of all transmission phases. Based on the analysis, we confirm that the proposed protocol outperforms the existing schemes in terms of outage probability, especially the scheme proposed in [6] which is asserted as outage-minimal in the same environment. We also obtain the closed form of DMT of the proposed protocol and show it approaches its upper bound in low multiplexing gain regime.

The rest of this paper is organized as follows. In Section II, we present system model of the proposed protocol. The description of our proposed protocol, VFD cooperative NOMA

This work was supported by the Basic Science Research Program through the NRF funded by the Ministry of Science and ICT (NRF-2016R1A2B4014834).

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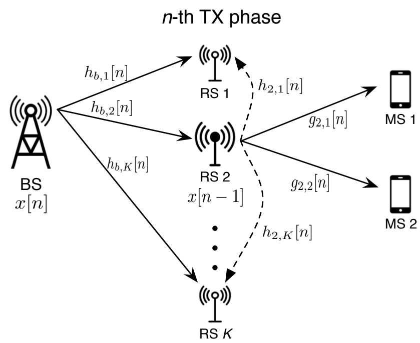

Fig. 1. System model

is given in Section III. Outage probability analysis using a Markov chain are provided in Section IV. In Section V, the proposed protocol is analyzed in asymptotic view. Numerical results are given in Section VI. In Section VII, conclusions are drawn.

II. SYSTEM MODEL

We consider a single-cell downlink network consisting of a single BS, two MSs, and K half-duplex decode-and-forward (DF) relay stations (RSs) as depicted in Fig. 1. Each node is assumed to be equipped with a single antenna, and we assume there is no direct link between the BS and two MSs as in [6]. In this paper, we assume a VFD operation at the RSs, in which a particular RS sends a packet while the other RSs receive a packet from the BS at the same time. Specifically, the number of total successive transmission phases is assumed to be N. The received signal at the k-th RS in the n-th transmission phase is given by

$$y_k^r[n] = h_{b,k}[n]x[n] + h_{j,k}[n]x[n-1] + z_k^r[n], \qquad (1)$$

where $x[n]$ and $h_{b,k}[n]$ denote the signal transmitted from the BS in the n-th transmission phase and the channel coefficient from the BS to the k-th RS in the n-th phase, respectively ($1 \leq k \leq K$, $1 \leq n \leq N$). Note that $x[0] = 0$. We assume that $h_{b,k}[n]$ is an i.i.d. complex Gaussian random variable, i.e., $h_{b,k}[n] \sim \mathcal{CN}(0,1)$ and $z_k^r[n]$ represents the thermal noise at the k-th RS in the n-th phase, which follows an i.i.d. complex Gaussian distribution, i.e., $z_k^r[n] \sim \mathcal{CN}(0, N_0)$. We assume that $\mathbb{E}[|x[n]|^2] = P$, and then the average signal-to-noise ratio (SNR) is given by $\rho = P/N_0$. In (1), without loss of generality, we assume that the j-th RS ($j \neq k$) is chosen to send the $(n-1)$-th packet to two MSs and $h_{j,k}$ denotes the channel coefficient from the j-th RS to the k-th RS, which is also an i.i.d. complex Gaussian random variable, i.e., $h_{j,k}[n] \sim \mathcal{CN}(0,1)$. In NOMA, the BS sends the superimposed signal that is given by $x[n] = \sqrt{a_1}s_1[n] + \sqrt{a_2}s_2[n]$, where $s_i[n]$ and $\sqrt{a}_i$ denote the desired signal of the i-th MS in the n-th phase and the power allocation coefficient for the i-th MS ($a_1 + a_2 = 1$), respectively.

Let $\mathcal{D}[n]$ be the index set of the RSs that successfully decode the n-th packet from the BS during the n-th transmission phase. Then, $|\mathcal{D}[n]|$ indicates the cardinality of the decoding set. A RS among the RSs included in $\mathcal{D}[n-1]$ is selected to send the decoded packet to two MSs in the n-th transmission phase. Thus, in (1), the j-th RS is assumed to succeed to decode the $(n-1)$-th packet from the BS, i.e., $j \in \mathcal{D}[n-1]$. The RS selection algorithm will be explained in the next section.

At the i-th MS, the received signal is given by $y_i^m[n] = g_{j,i}[n]x[n-1] + z_i^m[n]$, where $g_{j,i}[n]$ denotes the channel coefficient from the j-th RS to the i-th MS ($i = 1, 2$), which is an i.i.d. complex Gaussian random variable, i.e., $g_{j,i}[n] \sim \mathcal{CN}(0,1)$, and $x[n-1]$ denotes the $(n-1)$-th packet from the BS. At RSs, we also assume that $\mathbb{E}[|x[n-1]|^2] = P$ and $z_i^m[n]$ represents the thermal noise at the i-th MS in the n-th phase, which follows an i.i.d. complex Gaussian distribution, i.e., $z_i^m[n] \sim \mathcal{CN}(0, N_0)$.

We assume that two MSs are categorized not by channel quality but by different QoS requirements as in [6]. In this paper, perfect CSI at the receiver (CSIR) is assumed. Throughout this paper, the priority of the first MS is assumed to be higher than that of the second MS. Dynamic power allocation strategies in each transmission phase may improve the performance, but it is out of the scope of this paper. Hence, we assume that a_1 and a_2 are fixed over N transmission phases.

III. VIRTUAL FULL-DUPLEX COOPERATIVE NOMA

Basically, the BS broadcasts the superposed signals for two MSs in each transmission phase except for the last transmission phase (*i.e.,* $n = N$) in order to overcome the throughput loss due to half-duplex operation of RSs. Meanwhile, a selected RS among which RSs successfully decode the received packet from the BS in the previous transmission phase (*i.e.,* $k \in \mathcal{D}[n-1]$) forwards the packet to the MSs. Hence, $N-1$ packets are sent to two MSs from the BS during N transmission phases in the proposed VFD cooperative NOMA. In this section, we first investigate conditions for the successful decoding of the received packet at the RSs, and then describe the proposed relay selection algorithm.

A. Conditions for Successful Decoding at RSs

When the decoding set is empty, i.e., $|\mathcal{D}[n-1]| = 0$, the conditions for the k-th RS to successfully decode the received signals, $s_1[n]$ and $s_2[n]$, at the n-th transmission phase are given by represented as

$$\text{(C1)} \; : \; \log\left(1+\frac{a_1\,|h_{b,k}[n]|^2}{a_2\,|h_{b,k}[n]|^2+1/\rho}\right) \geq \frac{NR_1}{N-1}, \qquad (2)$$

$$\text{(C2)} \; : \; \log\left(1+a_2\rho|h_{b,k}[n]|^2\right) \geq \frac{NR_2}{N-1}, \qquad (3)$$

where R_1 and R_2 denote the target rate for the first and second MSs, respectively. Note that (2) and (3) represents the conditions of successful decoding at RSs when there is no inter-RS interference signals. At the first transmission phase, these conditions are used at RSs since $|\mathcal{D}[0]| = 0$.

When the inter-RS interference exists, i.e., $|\mathcal{D}[n-1]| \neq 0$, all RSs except for the selected RS suffer from the interference signals from the selected RS at the n-th transmission phase. In this case, the conditions for the successful decoding at the RS depends on whether it is included in the previous decoding set or not. If the k-th RS is not selected to relay signals to MSs and belongs to the previous decoding set, i.e., $k \in \mathcal{D}[n-1]$, then the conditions for the successful decoding are the same as (2) and (3) because it already has the interference signals from the selected RS and knows the channel coefficient from the selected RS to itself by the assumption of local CSI.

On the other hand, if $k \notin \mathcal{D}[n-1]$, then the k-th RS tries to perform joint decoding of both the n-th desired signal from the BS and the $(n-1)$-th interference signal from the selected RS. In this case, a multiple-access channel (MAC) becomes formed at the RSs except for the selected RS, consisting of the BS (superposed desired signals) and the selected RS (superposed interference signals). In order to obtain the conditions for successful decoding of the n-th transmission phase at the k-th RS such that $k \notin \mathcal{D}[n-1]$, we need to investigate the achievable rate region of the MAC channel consisting of 4 users who send $\sqrt{a_1}s_1[n-1]$, $\sqrt{a_2}s_2[n-1]$, $\sqrt{a_1}s_1[n]$, and $\sqrt{a_2}s_2[n]$, respectively. Note that $\sqrt{a_1}s_1[n-1]$ and $\sqrt{a_2}s_2[n-1]$ are transmitted through $h_{j,k}[n]$, whereas $\sqrt{a_1}s_1[n]$ and $\sqrt{a_2}s_2[n]$ are sent through $h_{b,k}[n]$ for RS k as a MAC channel receiver. Let $m_1 = a_1 |h_{j,k}[n]|^2$, $m_2 = a_2 |h_{j,k}[n]|^2$, $m_3 = a_1\rho |h_{b,k}[n]|^2$, and $m_4 = a_2 |h_{b,k}[n]|^2$. Also, R_i' is assumed to be R_1 and R_2 when i is and odd and even number, respectively. Base on the rate region of a MAC channel, in the joint decoding case for the relays not in the decoding set except for the selected relay ($k \notin \mathcal{D}[n], k \neq j$), the conditions of decoding are represented as the intersection of the all possible cases of 15:

$$\log\left(1+\rho\sum_{i\in\mathcal{M}} m_i\right) \geq \frac{N\sum_{i\in\mathcal{M}} R_i'}{N-1}, \forall\mathcal{M}\subset\{1,2,3,4\}. \quad (4)$$

B. Relay Selection

Of relays in the decoding set in the $(n-1)$-th phase, which decoded the received signals successfully based on (2) and (4), a single relay is selected in every phase to forward (broadcast) the received signals, known as opportunistic reactive DF relaying. For relay selection, we propose the two-stage relay selection criterion. At the first stage, we define the subset $\mathcal{S}[n]$ based on cardinality of the decoding set in the previous phase under the following conditions for $2 \leq n \leq N$: $\mathcal{S}[n] = \left\{k \in \mathcal{D}[n-1] \middle| \log\left(1+\frac{a_1|g_{k,i}[n]|^2}{a_2|g_{k,i}[n]|^2+\frac{1}{\rho}}\right) \geq \frac{NR_1}{N-1}, \forall i = 1,2\right\}$. In the sequel, relay j is selected in the n-th phase by the following criterion:

$$j = \arg\max_{k\in\mathcal{S}[n]} |g_{k,2}[n]|^2, \; 2 \leq n \leq N. \quad (5)$$

Note that (5) focuses on channel gain of MS2 since $\mathcal{S}[n]$ already picked up RSs which are capable of decoding $s_1[n]$ before relay selection. Finally, the selected relay j forwards the received signals to the users in the n-th phase.

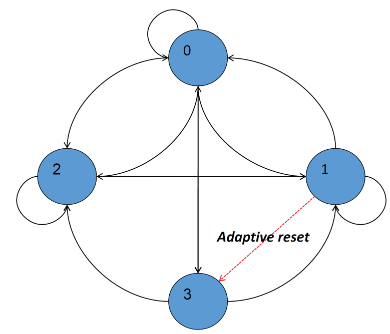

Fig. 2. An example of adaptive reset: The case for $K = 3$ and $N_c = 1$

The main difference of relay selection in this paper and [6] is whether the subset $\mathcal{S}[n]$ includes the relays decoding $s_2[n-1]$ successfully or not. That is, the two-stage relay selection scheme in [6] can select the relays failing decoding $s_2[n-1]$while the proposed protocol can select the relays decoding both $s_1[n-1]$ and $s_2[n-1]$ successfully.

C. Adaptive Reset

In order to improve the reliability of the proposed protocol, we consider adaptive reset. The main idea of adaptive reset, similarly considered in [10], is to allow refreshing and restarting of the proposed protocol even before $\mathcal{S}[n]$ is empty. Let N_c be the cardinality of the decoding set which determines the time to restart the transmission protocol. For example, $N_c = 1$ means that the protocol restarts when $|\mathcal{S}[n]| \leq 1$, as shown in Fig. 2. The dotted red arrow from state 1 (N_c) to state 3 (K) corresponds to adaptive reset, whereas the arrow does not exist if adaptive reset is not adopted. As a result, adaptive reset secures diversity gain even in the high multiplexing gain regime, which will be justified via asymptotic analysis in Section V.

IV. Outage Probability Analysis

In this section, we analyze the proposed protocol in terms of outage probability. In order to compute outage probability in the n-th phase, we should consider the $(n-1)$ states of the subset in all previous phases. However, outage probability in the n-th phase follows Markovity, where it depends on only the $(n-1)$-th phase. Owing to the Markovity, outage probability in the n-th phase denoting $\Pr\{\mathcal{O}_n\}$ is given by

$$\begin{aligned}\Pr\{\mathcal{O}_n\} &= \sum_{t=0}^{K}\Pr\left\{\mathcal{O}_n \middle| |\mathcal{S}[l]| = t, l = 2,\dots,n\right\} \\ &\quad\times \Pr\{|\mathcal{S}[l]| = t, l = 2,\dots,n\} \\ &\overset{(a)}{=} \sum_{t=0}^{K}\Pr\left\{\mathcal{O}_n \middle| |\mathcal{S}[n]| = t\right\}\Pr\{|\mathcal{S}[n]| = t\} \\ &= \sum_{t=0}^{K}\left(\Pr\left\{|g_{j,2}[n]|^2 < \frac{2^{\frac{NR_2}{N-1}}-1}{a_2\rho}\right\}\right)^t \Pr\{|\mathcal{S}[n]| = t\}\end{aligned}$$

where (a) follows from the Markovity. Outage probability in the n-th phase is simplified but it is still intractable because outage probability in each phase depends on cardinality of $\mathcal{S}[n-1]$ in the previous phase. Thus, we adopt Markov chain in order to obtain the closed form of $\Pr\{|\mathcal{S}[n]|=t\}$ so that outage analysis is tractable which means $\Pr\{\mathcal{O}_n\} = \Pr\{\mathcal{O}\}$ in steady state.

Theorem 1: When there are half-duplex single-antenna K relays in the absence of direct links between the BS and users with single antenna at each, the outage probability of the proposed protocol is given by

$$\Pr\{\mathcal{O}\} = \sum_{t=0}^{K} \left(1 - \exp\left(-\frac{2^{\frac{NR_2}{N-1}}-1}{a_2\rho}\right)\right)^t \pi_t \qquad (6)$$

where $\pi_t, \forall t \in \{0, \dots, K\}$ denotes an element of a $(K+1)$-dimensional stationary distribution vector $\boldsymbol{\pi}(= [\pi_0, \pi_1, ..., \pi_K],\ \sum_{i=0}^{K} \pi_i = 1)$ for the Markov chain and $\exp(x) = e^x$.

Proof: Consider the Markov chain whose states are related to cardinality of the subset $\mathcal{S}$, defined by

$$\mathbf{P}_K = \begin{pmatrix} P_{0,0} & \dots & P_{0,K-1} & P_{0,K} \\ \vdots & \vdots & \vdots & \vdots \\ P_{0,0} & \dots & P_{0,K-1} & P_{0,K} \\ P_{K-N_c,0} & \dots & P_{K-N_c,K-1} & 0 \\ \vdots & \dots & \vdots & \vdots \\ P_{K,0} & \dots & P_{K,K-1} & 0 \end{pmatrix} \qquad (7)$$

where $P_{i,j}$ is the transition probability from state i to j which more specifically means $\Pr\{|\mathcal{S}[l-1]| = i \rightarrow |\mathcal{S}[l]| = j\}$ for the arbitrary phase, $l = 2, \dots, N$. Note that the first N_c columns are the same since they are transition probabilities from the initial state due to refreshing and the elements except for first $N_c + 1$ $P_{0,K}$s in the last column is zero since a selected relay always forwards the received signals and thus the cardinality of $\mathcal{S}$ cannot be K except for when adaptive reset is adopted.

Each element of $\mathbf{P}_K$ can be formulated as (8) shown on the top of the next page, where p_o and $p_{o_{\text{int}}}$ are the probability that a relay does not belong to $\mathcal{S}$ in the absence (SIC or silent relays case) and presence (joint decoding case) of inter-relay interference signals, respectively. We can easily confirm that the Markov chain reaches a steady state as in [10]. That is, $\boldsymbol{\pi}\mathbf{P}_K = \boldsymbol{\pi}$ holds. Then, solving $\boldsymbol{\pi}\mathbf{P}_K = \boldsymbol{\pi}$ subject to $\sum_{i=0}^{K} \pi_i = 1$, we can obtain the closed form of $\Pr\{|\mathcal{S}[n]|=t\}$ exploiting property of the stationary distribution since $\Pr\{|\mathcal{S}[n]|=t\} = \pi_t$ in steady state. $|h_{j,2}[n]|^2$ follows exponential distribution. Finally, we can attain (6). ■

Corollary 1: When $K = 3$,

$$\Pr\{\mathcal{O}\} = \sum_{t=0}^{3} \left(1 - \exp\left(-\frac{2^{\frac{NR_2}{N-1}}-1}{a_2\rho}\right)\right)^t \pi_t, \qquad (9)$$

where $\pi_0 = \frac{\{P_{2,0}P_{0,2}+P_{0,0}(1-P_{2,2})\}\pi_2+(P_{3,0}P_{0,2}-P_{3,2}P_{0,0})\pi_3}{P_{0,2}}$, $\pi_1 = 1 - \pi_0 - \pi_2 - \pi_3$, $\pi_2 = \frac{P_{0,2}-P_{3,2}(P_{0,0}+P_{1,1}+P_{0,2})}{(1+P_{0,3})(1-P_{2,2})-P_{0,2}+P_{3,2}(P_{0,0}+P_{1,1}+P_{0,2})}$ and $\pi_3 = \frac{P_{0,3}(1-\pi_2)}{1+P_{0,3}}$, where $\mathcal{T} = P_{0,2} + P_{0,3}P_{3,2} - P_{1,2} - P_{1,2}P_{0,3}$ and $\mathcal{C} = 1 - P_{1,1} - P_{0,3}P_{1,1} + P_{0,1} + P_{0,3} + P_{0,3}P_{3,1}$.

Proof: Plugging $K = 3$ in (6) and solving $\boldsymbol{\pi}\mathbf{P}_3 = \boldsymbol{\pi}$, we can easily obtain (9). ■

Remark 1: It can be empirically confirmed that $\mathbf{P}_K^n\mathbf{P}_K = \mathbf{P}_K^n$ holds within single-digit n on average. That is, the Markov chain reaches a steady state after a negligible number of phases compared to sufficient N, and it validates the snapshop approach for outage probability analysis of the proposed protocol.

V. Asymptotic Analysis

In this section, we asymptotically analyze the proposed protocol with respect to ρ, N and K based on outage probability analysis. Basically, we exploit DMT [11] the main metric. For DMT analysis, note that multiplexing gain and diversity gain are defined, respectively, by $r = \lim_{\rho\to\infty} \frac{\log R_i(\rho)}{\log \rho}$ and $d = \lim_{\rho\to\infty} -\frac{\log \mathrm{P}_{\text{out}}(\rho)}{\log \rho}$ where $R_i(\rho)$ represents the transmission rate of MS i, and P_{out} denotes outage probability. Let us define $f(\rho) \doteq \rho^v$ if $\lim_{\rho\to\infty} \frac{\log(f(\rho))}{\log \rho} = v$. a^+ denotes $\max(0, a)$ for any real value a, and $\log(\cdot)$ denotes the base-2 logarithm. Focusing on DMT performance, substitute R_i with $r_i \log \rho$ and define $\pi_t \doteq \tilde{\pi}_t$ where $\tilde{\pi}_t$, a element of $\tilde{\boldsymbol{\pi}}$, is the dominant scale of π_t. Then, the outage probability scales like

$$\Pr\{\mathcal{O}\} \doteq \sum_{t=0}^{K} \left(\frac{2^{\bar{r}_2 \log \rho}}{a_2\rho}\right)^t \tilde{\pi}_t \doteq \sum_{t=0}^{K} \rho^{-t(1-\bar{r}_2)} \cdot \tilde{\pi}_t, \qquad (10)$$

where $\bar{r}_i$ is effective multiplexing gain of MS i ($\bar{r}_i = cr_i$), determined by the further asymptotic analysis shown later. In order to represent $\tilde{\pi}_t$, it is necessary to formulate the dominant scale of the outage probabilities conditioned on the decoding status of RS k in the previous phase. For the case when RS k succeeded decoding in the previous phase, the outage probability for RS k is the same as the case in the absence of inter-relay interference due to SIC. Hence, we have

$$\begin{aligned} p_o = 1 - &\Pr\left(|h_{b,k}[n]|^2 > \frac{2^{\bar{r}_2 \log \rho}}{\rho}\right) \\ &\times \Pr\left(|h_{b,k}[n]|^2 > \frac{2^{\bar{r}_1 \log \rho} - 1}{\{a_1 - (2^{\bar{r}_1 \log \rho})a_2\}\rho}\right) \\ &\times \Pr\left(|g_{k,1}[n]|^2 > \frac{2^{\bar{r}_1 \log \rho} - 1}{\{a_1 - (2^{\bar{r}_1 \log \rho})a_2\}\rho}\right) \\ &\times \Pr\left(|g_{k,2}[n]|^2 > \frac{2^{\bar{r}_1 \log \rho} - 1}{\{a_1 - (2^{\bar{r}_1 \log \rho})a_2\}\rho}\right) \\ = 1 - &\exp\left\{-\frac{2^{\bar{r}_2 \log \rho}}{\rho} - 3\frac{2^{\bar{r}_1 \log \rho} - 1}{\{a_1 - (2^{\bar{r}_1 \log \rho} - 1)a_2\}\rho}\right\} \\ \doteq &\frac{2^{\bar{r}_2 \log \rho}}{\rho} + 3\frac{2^{\bar{r}_1 \log \rho}}{\{a_1 - (2^{\bar{r}_1 \log \rho})a_2\}\rho} \\ \doteq &\rho^{\bar{r}_2 - 1} + 3\frac{\rho^{\bar{r}_1}}{\rho - \rho^{\bar{r}_1 + 1}} \doteq \rho^{-(1-\bar{r}_2)}, \end{aligned} \qquad (11)$$

since $\frac{\rho^{\bar{r}_1}}{\rho - \rho^{\bar{r}_1+1}}$ goes to zero when ρ is sufficiently large. Meanwhile, assuming P_{MAC} denotes the probability that (4) holds,

$$p_{o_{\text{int}}} = 1 - P_{\text{MAC}}\Pr\left(|g_{k,1}[n]|^2 > \frac{2^{\bar{r}_1 \log \rho} - 1}{\{a_1 - (2^{\bar{r}_1 \log \rho})a_2\}\rho}\right)$$

$$P_{i,j}=\begin{cases}\binom{K}{j}p_o^{K-j}(1-p_o)^j, & \text{if } i=0\\ \sum_{x=0}^{\min(i,j)}\binom{i-1}{x}p_o^{i-1-x}(1-p_o)^x\binom{K-i}{j-x}p_{o_{\text{int}}}^{K-i-j+x}(1-p_{o_{\text{int}}})^{j-x}, & \text{if } i\neq 0\end{cases}, \tag{8}$$

$$\times \Pr\left(|g_{k,2}[n]|^2>\frac{2^{\bar{r}_1\log\rho}-1}{\{a_1-(2^{\bar{r}_1\log\rho})a_2\}\rho}\right)$$
$$\doteq \rho^{-(1-\bar{r}_i)}+\rho^{-2(1-2\bar{r}_i)}+\rho^{-(1-\sum_l\bar{r}_l)}+\rho^{-2(1-\bar{r}_1-2\bar{r}_2)}$$
$$+\rho^{-2(1-2\bar{r}_1-\bar{r}_2)}+\rho^{-2(1-2\sum_l\bar{r}_l)}+2\frac{\rho^{\bar{r}_1}}{\rho-\rho^{\bar{r}_1+1}}$$
$$\doteq\rho^{-\min(1-\bar{r}_i,2(1-2\bar{r}_i),1-\sum_l\bar{r}_l,2(1-\bar{r}_1-2\bar{r}_2),2(1-2\bar{r}_1-\bar{r}_2),2(1-2\sum_l\bar{r}_l))} \tag{12}$$

for all $i=1,2$ where $\sum_l\bar{r}_l=\bar{r}_1+\bar{r}_2$.

However, it is intractable and complicated to attain the dominant scale of stationary probability of the Markov chain, $\tilde{\pi}_t$. Hence, as assumed in [12], [13] in order to effectively characterize DMT for each individual multiplexing gain, we deal with symmetric multiplexing gain case ($r_1=r_2=r$). It is noted that the symmetric multiplexing case is acceptable since multiplexing gain of each user can be the same though achievable rat of each user is different for NOMA, *i.e.,* we can say $r_1=r_2=r$ if $R_1>R_2$ and $R_1\doteq R_2\doteq\rho^r$. Since $r_1=r_2=r$ implies $\bar{r}_1=\bar{r}_2=\bar{r}$, under the assumption of symmetric multiplexing gain, (11) and (12) are reduced to

$$p_o\doteq\rho^{-\min(1-\bar{r})},\quad p_{o_{\text{int}}}\doteq\rho^{-\min(1-2\bar{r},2(1-4\bar{r}))}, \tag{13}$$

respectively.

Theorem 2: When each node has one antenna, the DMT of the proposed protocol for N_c is given by (14) on the top of the next page.

Proof: Plugging (13) into (7), we will obtain $\tilde{\boldsymbol{\pi}}$ which $\mathbf{P}_K\doteq\mathbf{P}_K^n\mathbf{P}_K$ holds. In order to find the dominant scale of converged transition probability, three mainly different multiplexing gain region should be classified since $p_{o_{\text{int}}}\doteq\rho^{-(1-2\bar{r})}$ for $0\le\bar{r}\le\frac{1}{6}$, $p_{o_{\text{int}}}\doteq\rho^{-2(1-4\bar{r})}$ for $\frac{1}{6}<\bar{r}\le\frac{1}{4}$, and $p_{o_{\text{int}}}\doteq\rho^0$ for $\frac{1}{4}<\bar{r}$. Computations of $\mathbf{P}_K^n$ is similar to the way in Appendix A of [10], and we skip the details of matrix computations due to the page limit. Consequently, we have

$$\tilde{\boldsymbol{\pi}}\doteq\Big[\rho^{-((K-1)-K\bar{r})},\rho^{-((K-2)-(K-1)\bar{r})},\rho^{-((K-3)-(K-2)\bar{r})},$$
$$\cdots,\rho^0,\rho^{-((K-1)-K\bar{r})}\Big]^T \text{ for } 0\le\bar{r}\le 1/6 \tag{15}$$

$$\tilde{\boldsymbol{\pi}}\doteq\min_{i=1,\dots,K-1-N}\tilde{\boldsymbol{\pi}}^{(i)},\ \text{for } 1/6<\bar{r}\le 1/4, \text{ where}$$
$$\tilde{\boldsymbol{\pi}}^{(i)}=\Big[\rho^{-\{(K-i-1)(1-\bar{r})+(i^2+i)(1-4\bar{r})\}},$$
$$\rho^{-\{(K-i-2)(1-\bar{r})+(i^2+i)(1-4\bar{r})\}},\dots,\rho^{-2(1-4\bar{r})},\rho^0,$$
$$\rho^{-\{(K-i-1)(1-\bar{r})+(i^2+i)(1-4\bar{r})\}}\Big], \tag{16}$$

$$\tilde{\boldsymbol{\pi}}\doteq\Big[\rho^{-N_c(1-\bar{r})},\cdots,\rho^{-(1-\bar{r})},\rho^0,\cdots,\rho^0\Big]\text{for } 1/4<\bar{r}. \tag{17}$$

For $0\le\bar{r}\le\frac{1}{4}$, since ρ^0 is only in the K-th element of $\tilde{\boldsymbol{\pi}}$, $\lim_{\rho\to\infty}\tilde{\boldsymbol{\pi}}\approx[0\ 0\ \cdots\ 0\ 1\ 0]$. That is, $\lim_{\rho\to\infty}\pi_{K-1}\approx 1$ and $\lim_{\rho\to\infty}\pi_t\approx 0,\ \forall t\neq K-1$ where π_t is the steady state probability of state t corresponding to $(t+1)$-th element of $\boldsymbol{\pi}$. It implies that no refreshing is needed in asymptotic region, thus $\bar{r}$ for $0\le\bar{r}\le\frac{1}{4}$ Note that two limits about n and ρ are independent so that the order of limits is irrelevant to the DMT results throughout this paper. Meanwhile, for $\frac{1}{4}<\bar{r}$, since ρ^0 appears in the last $K-N_c+1$ elements, $\lim_{\rho\to\infty}\tilde{\boldsymbol{\pi}}\approx[0\ 0\ \cdots\ c_{N_c}\ \cdots\ c_{K-1}\ c_K]$ and it is easily shown that all the coefficients of ρ^0 are the same. Therefore, $c_{N_c}=\cdots=c_K=\frac{1}{K-N_c+1}$ and $\bar{r}=\frac{K-N_c+1}{K-N_c}r$ for $\frac{1}{4}<\bar{r}$ because adaptive reset occurs when $|\mathcal{S}[n]|\le N_c$.

Replacing $\tilde{\pi}_t$ in (10) by the elements of $\tilde{\boldsymbol{\pi}}$, we can obtain (14). ■

Corollary 2: The proposed protocol approximately achieves the upper bound of DMT For $0<r\le\frac{1}{4}$ as K increases given the optimal value of $N_c=K-2$.

Proof: For $0<r\le\frac{1}{4}$, since $\lim_{\rho\to\infty}\pi_{K-1}\approx 1$, $N_c=K-2$ maximizes $d(r,K,N_c)$. Then, we have

$$\lim_{K\to\infty}d(r,K,N_c)|_{N_c=K-2}=\lim_{K\to\infty}\min\{(K-1)-Kr,K-(K+6)r\}$$
$$\simeq\min\{(K-1)(1-r),K(1-r)\}=(K-1)(1-r)=d_u(r,K). \tag{18}$$

Note that the upper bound reflects that a selected relay always cannot contribute to diversity gain. ■

VI. Numerical Results

In this section, we evaluate the proposed protocol and compare it with the referential schemes including the protocol in [6] in terms of outage probability and DMT as metrics of non-asymptotic and asymptotic analysis, respectively. 'The proposed', 'Two-stage RS', and 'Successive' stand for the proposed protocol in this paper, the protocol in [6], and successive transmission based on relay selection dealing with inter-relay interference as noise, respectively.

Fig. 3 characterizes outage probability of the schemes when $K=10$ according to specific examples of a_1. R_1 and R_2 are assumed to be 0.5 bit/s/Hz and 2 bit/s/Hz, respectively. Period of refreshing successive transmission regardless of the status of decoding set, N, determining spectral efficiency, is set to 20 for 'The proposed' and 'Successive', respectively. When $K=10$ as shown in Fig. 3, the proposed technique outperforms both the two-stage RS technique and the successive relaying technique especially for various values of a_1. It is worth noting that the *optimal* a_1, resulting in the lowest outage probability, is different for the proposed technique and the two-stage RS technique. For example, the outage performance of the proposed technique when $a_1=0.5$ is better than the case when $a_1=0.75$ and $a_1=0.95$. However, the outage performance of the two-stage RS selection algorithm

$$d(r,K,N_c)=\begin{cases} K-1-Kr, & \text{if} \quad 0 \le r \le \frac{1}{6} \\ \min\limits_{i=1,\ldots,K-1-N_c} \left\{(K-1-i)(1-r)+(i^2+i)(1-4r)\right\}, & \text{if} \quad \frac{1}{6} < r \le \frac{1}{4} \\ N_c\left(1-\frac{K-N_c+1}{K-N_c}r\right)^{+}, & \text{if} \quad \frac{1}{4} < r \le 1 \end{cases} \tag{14}$$

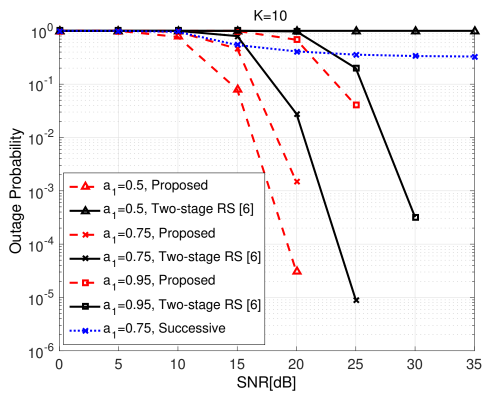

Fig. 3. Outage probability when $K = 10$ and $N_c = 0$ for $R_1 = 0.5$ bit/s/Hz and $R_2 = 2$ bit/s/Hz.

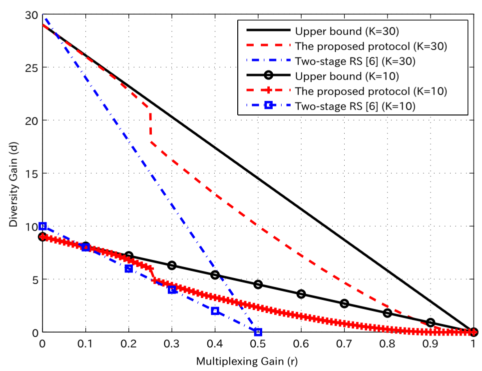

Fig. 4. DMT when $K = 10,\ 30$.

with $a_1 = 0.75$ is better than the case when $a_1 = 0.5$ and $a_1 = 0.95$.

Fig. 4 compares DMT of the proposed protocol and two-stage relay selection when $K = 10$ and 30. The upper bounds are plotted based on (18). In the both $K = 10$ and 30 cases, the proposed protocol outperforms the two-stage relay selection in terms of DMT except for the low multiplexing gain regime. It comes from the inherent difference of diversity gain between the proposed protocol and two-stage relay selection, which a single relay in the proposed protocol is always excluded to be selected for VFD operation. For $0 \le r \le \frac{1}{4}$, DMT of proposed protocol approaches the upper bound, and it validates *Corollary 2*.

VII. Conclusions

In this paper, we proposed the protocol that can operate VFD with multiple half-duplex DF RSs leveraging dynamic inter-relay interference management in the downlink cooperative NOMA framework. We also introduced the two-stage relay selection as well as adaptive reset. We analyzed the proposed protocol in terms of outage probability and DMT, and obtain the closed form of the both metrics. Numerical results validated the analysis, and we confirmed that outage probability of the proposed protocol surpassed that of the conventional cooperative NOMA in the same environment. Also, the proposed protocol almost outperformed the conventional cooperative NOMA in terms of DMT.

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